# Math 115

# Spring 2019

# Lecture 18

$? \; a^2 + b^2 = c^2 \; ?$

$y = mx + b \quad ? \quad d = rt$

## Operations with polynomial

1) Distribution

$3x(4x^2 - 5x + 1)$

$= 3x \cdot 4x^2 - 3x \cdot 5x + 3x \cdot 1$

$= \boxed{12x^3 - 15x^2 + 3x}$

Trinomial
Deg. = 3
Lead. Coef. = 12
No Constant

$-5x^2(3x^3 - 4x^2 + 7x - 1)$

$= -5x^2 \cdot 3x^3 + 5x^2 \cdot 4x^2 - 5x^2 \cdot 7x + 5x^2 \cdot 1$

$= \boxed{-15x^5 + 20x^4 - 35x^3 + 5x^2}$

D = 5
L.C. = −15

Distribute & Simplify

$7x(2x^3 - 4x + 1) - 2x^2(7x^2 - 3x - 14) - 20$

$= 14x^4 - 28x^2 + 7x - 14x^4 + 6x^3 + 28x^2 - 20$

$= 7x + 6x^3 - 20 = \boxed{6x^3 + 7x - 20}$

Trinomial
D = 3
L.C. = 6
Constant = -20

Multiplication with polynomials using FOIL method

First ones
Outside ones
Inside ones
Last ones

$(3x + 5)(4x - 1)$

F: $3x \cdot 4x$
O: $3x \cdot 1$
I: $5 \cdot 4x$
L: $5 \cdot 1$

$= 3x \cdot 4x - 3x \cdot 1 + 5 \cdot 4x - 5 \cdot 1$

$= 12x^2 - 3x + 20x - 5 = \boxed{12x^2 + 17x - 5}$

Like terms

Trinomial,
D = 2, L.C. = 12, Const = -5

Multiply using Foil method:

$(10x + 3)(10x - 3)$

Conjugates

$(A + B)(A - B)$

$= 10x \cdot 10x - 10x \cdot 3 + 3 \cdot 10x - 3 \cdot 3$

$= 100x^2 - 30x + 30x - 9$

$= \boxed{100x^2 - 9}$

Binomial
D=2, L.C.=100, Const=-9

Multiply by FOIL method:

$(4x^2 - 5)(3x^2 - 4)$

$= 12x^4 - 16x^2 - 15x^2 + 20$

$= \boxed{12x^4 - 31x^2 + 20}$

Trinomial
D=4, L.C.=12, Const=20

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Use the concept of FOIL to multiply

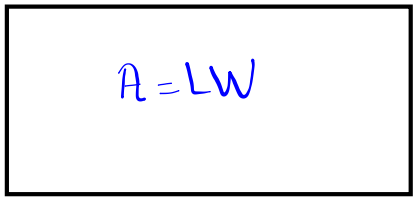
$(3x + 2)(9x^2 - 6x + 4)$

Binomial
D=3, L.C.=27

$= 27x^3 - 18x^2 + 12x + 18x^2 - 12x + 8 = \boxed{27x^3 + 8}$

Const=8

Find the area:




$x^2+4x-1$

$2x^2-3x+4$

$A = LW$

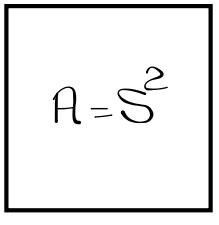
$= (2x^2 - 3x + 4)(x^2 + 4x - 1)$

$= 2x^4 + 8x^3 - 2x^2 - 3x^3 - 12x^2 + 3x + 4x^2 + 16x - 4$

$= 2x^4 + 5x^3 - 10x^2 + 19x - 4$

Polynomial
D=4, LC=2, Const=-4

Find the area




$3x+5$

$A = S^2$

$= (3x+5)^2$

$= (3x+5)(3x+5)$

$= 9x^2 + 15x + 15x + 25$

$= 9x^2 + 30x + 25$

Trinomial
D=2, LC=9, Const=25

$(A+B)^2 \neq A^2 + B^2$

$(A+B)^2 = (A+B)(A+B)$

Simplify:

① $4(2x-3y)-2(4x+6y)$

$= 8x - 12y - 8x - 12y = \boxed{-24y}$

Monomial
D = 1
C = −24

② $x(x^2-5x+3)+2(x^3-3x^2-4x-5)$

$= x^3 - 5x^2 + 3x + 2x^3 - 6x^2 - 8x - 10$

$= \boxed{3x^3 - 11x^2 - 5x - 10}$

Polynomial
D = 3
LC = 3
Const = −10

③ $(3x+5y)(4x-2y)$

$= 12x^2 - 6xy + 20xy - 10y^2$

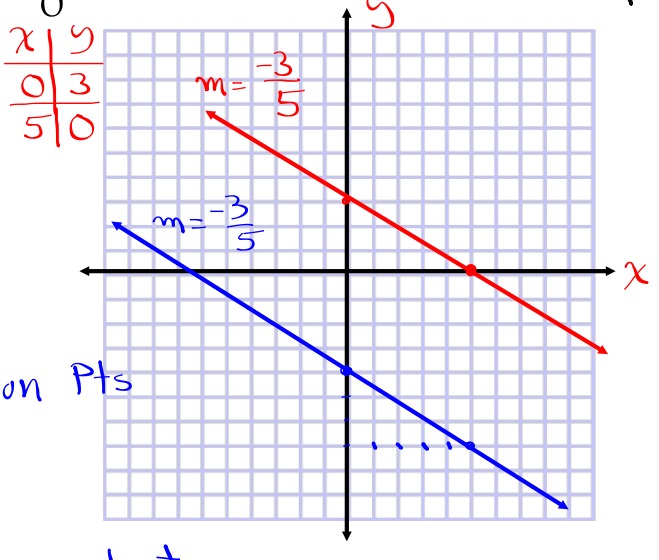
$= \boxed{12x^2 + 14xy - 10y^2}$

Trinomial
D = 2
L.C. = 12
NO Constant

Solve by graphing, then discuss System & equations:

$$\begin{cases} 3x + 5y = 15 \\ y = \frac{-3}{5}x - 4 \end{cases}$$

| x | y |
|---|---|
| 0 | 3 |
| 5 | 0 |




Parallel lines

No intersection Pts

No Soln.

System: Inconsistent

Equations: Independent

Solve by addition method, then discuss system & equations:

$$2\begin{cases} 2x - 5y = 10 \\ -4x + 10y = 15 \end{cases} \Rightarrow \begin{cases} 4x - 10y = 20 \\ -4x + 10y = 15 \end{cases}$$

$0 = 35$

False

System: Inconsistent

Eqns: Independent

$\emptyset$

Scientific Notation

It is used for very large or very small numbers.

$N \times 10^{n}$ ← n: any integer

$1 \leq N < 10$

$1.5 \times 10^{15}$

$8.2 \times 10^{-12}$

$4.37 \times 10^{32}$

$7.1 \times 10^{-18}$

Write 4500000000 in S.N.

9 times

$4.5 \times 10^{9}$

Write .00000000000000825 in S.N.

14 times

$8.25 \times 10^{-14}$

Simplify $(2.5 \times 10^{16}) \cdot (1.8 \times 10^{10})$

$= 4.5 \times 10^{26}$

Simplify $\dfrac{6.5 \times 10^{25}}{2 \times 10^{8}}$

$= 3.25 \times 10^{17}$

Simplify $(7.5 \times 10^{-21}) \cdot (6.2 \times 10^{8})$

$= 46.5 \times 10^{-13}$

$= 4.65 \times 10^{1} \times 10^{-13} = 4.65 \times 10^{-12}$

Simplify $\dfrac{1.5 \times 10^{-23}}{6 \times 10^{10}}$ $\quad -23-10$

$= .25 \times 10^{-33}$

$= 2.5 \times 10^{-1} \times 10^{-33} = \boxed{2.5 \times 10^{-34}}$

Back to polynomials:

Special Products

① $(A+B)^2 = (A+B)(A+B)$

$= A^2 + AB + BA + B^2$

$= A^2 + AB + AB + B^2$

$\boxed{(A+B)^2 = A^2 + 2AB + B^2}$

Ex: $(x+8)^2 = x^2 + 2 \cdot x \cdot 8 + 8^2$

$= \boxed{x^2 + 16x + 64}$

$(2x+5)^2 = (2x)^2 + 2(2x)(5) + (5)^2$

$= \boxed{4x^2 + 20x + 25}$

Trinomial
D=2, L.C.=4
Const=25

$(3x^2+7y^3)^2 = (3x^2)^2 + 2(3x^2)(7y^3) + (7y^3)^2$

$= \boxed{9x^4 + 42x^2y^3 + 49y^6}$

Trinomial, D=6, LC=49

Simplify $(8x^5 + 4x^3)^2$

$= (8x^5)^2 + 2(8x^5)(4x^3) + (4x^3)^2$

$= \boxed{64x^{10} + 64x^8 + 16x^6}$

Trinomial
D=10
LC=64
No Constant

$$(A-B)^2 = (A-B)(A-B)$$

$$= A^2 - AB - BA + B^2$$

$$= A^2 - AB - AB + B^2$$

$$\boxed{(A-B)^2 = A^2 - 2AB + B^2}$$

Ex:

$$(x-3)^2 = (x)^2 - 2(x)(3) + (3)^2$$

$$= \boxed{x^2 - 6x + 9}$$

Trinomial
D=2, LC=1,
Const=9

$$(4x^2-5)^2 = (4x^2)^2 - 2(4x^2)(5) + (5)^2$$

$$= 16x^4 - 40x^2 + 25$$

Trinomial, D=4, L.C.=16,
Const=25

$$(6x^5 - 7y^3)^2 = (6x^5)^2 - 2(6x^5)(7y^3) + (7y^3)^2$$

$$= \boxed{36x^{10} - 84x^5y^3 + 49y^6}$$

Trinomial, D=10, LC=36,
No constant

$(11x^8 - 3x^3)^2$

$= (11x^8)^2 - 2(11x^8)(3x^3) + (3x^3)^2$

$= \boxed{121x^{16} - 66x^{11} + 9x^6}$

Trinomial, D=16, LC=121, No constant

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$(A+B)(A-B) = A^2 - AB + BA - B^2$

Conjugates $= A^2 - \cancel{AB} + \cancel{AB} - B^2$

$\boxed{(A+B)(A-B) = A^2 - B^2}$

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E $(x+6)(x-6) = (x)^2 - (6)^2$

Conjugates $= \boxed{x^2 - 36}$

$(5x-9)(5x+9) = (5x)^2 - (9)^2$

Conjugates $= \boxed{25x^2 - 81}$

$(10x^3 + 7y^2)(10x^3 - 7y^2) = (10x^3)^2 - (7y^2)^2$

Conjugates $= \boxed{100x^6 - 49y^4}$

Simplify

$(5x^2y^3 + 8x^3y)(5x^2y^3 - 8x^3y)$

Conjugates

$= (5x^2y^3)^2 - (8x^3y)^2$

$= 25x^4y^6 - 64x^6y^2$

$D = 10$ $\quad$ $D = 8$

$C = 25$ $\quad$ $C = -64$

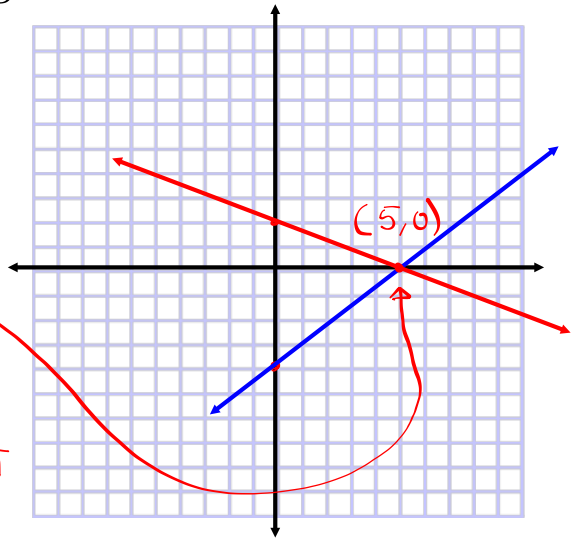
Binomial

$D = 10$

L.C. $= 25$

Solve by graphing, discuss system & eqns

$$\begin{cases} y = \frac{4}{5}x - 4 \\ 2x + 5y = 10 \end{cases}$$




Soln (5,0)

System : Consistent

Eqns : Independent

Solve by Subs., Discuss system & eqns:

$$\begin{cases} y = 2x - 5 \\ 2x + 3y = 11 \end{cases}$$

$2x + 3(2x - 5) = 11$

$2x + 6x - 15 = 11$

$8x = 26$

$x = \frac{26}{8}$ $\quad \boxed{x = \frac{13}{4}}$

$y = 2\left(\frac{13}{4_2}\right) - 5$

$= \frac{13}{2} - \frac{10}{2} = \frac{13-10}{2} = \frac{3}{2}$

$\boxed{y = \frac{3}{2}}$

Soln: $\left(\frac{13}{4}, \frac{3}{2}\right)$

System: Consistent

Equations: Independent.

Solve by addition method, discuss system & Equations.

$$3\begin{cases} 3x - 2y = 4 \\ 2x + 3y = 8 \end{cases} \Rightarrow \begin{cases} 9x - 6y = 12 \\ 4x + 6y = 16 \end{cases}$$

$13x = 28$

$\boxed{x = \frac{28}{13}}$

$$-2\begin{cases} 3x - 2y = 4 \\ 3(2x + 3y = 8) \end{cases}$$

$13y = 16$

$\boxed{y = \frac{16}{13}}$

$\left(\frac{28}{13}, \frac{16}{13}\right)$ see last example

Solve by Substitution, then discuss System & Equations:

$$\begin{cases} 3x - y = 4 \\ y = 3x - 4 \end{cases}$$

$3x - (3x - 4) = 4$

$3x - 3x + 4 = 4$

$4 = 4$

True

infinitely Many Solutions

System: Consistent

Equations: Dependent

| When we have | System | Equations |
|---|---|---|
| exactly one Solution | Consistent | independent |
| infinitely many Solutions | Consistent | Dependent |
| NO Solution | Inconsistent | Independent |

Work on SG 13 & 14 for Thursday.